THE TURBULENT DIFFUSION COMBUSTION OF A GAS IN A CYLINDRICAL CHAMBER

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The turbulent diffusion combustion of a gas jet in a surrounding air flow bounded by the walls of a cylindrical chamber is studied theoretically and experimentally. A generalized universal equation is obtained for the coefficient of completeness of burning in the chamber as a function of the parameters of the geometry and modes of operation

In many types of combustion chamber and industrial furnace the gaseous fuel and the gaseous oxidant are introduced separately so that turbulent diffusion combustion takes place in the chamber. Whereas the fundamental laws of turbulent diffusion jets developing in a volume full of fluid or in a surrounding flow have been studied in considerable detail [1-6], the problem of turbulent combustion in a chamber remains essentially open.

In this paper we make an attempt to discuss the simplest case – a single diffusion turbulent jet in a cylindrical combustion chamber.

1. It is assumed that combustion begins immediately at the end of the pipe, i e., that the diffusion jet is safely stabilized. We shall also assume that in turbulent diffusion combustion also, the chemical reactions take place in an infinitely thin front separating the combustion chamber into two regions, in each of which the fuel concentration or the oxidant concentration is zero. Thus, we consider a scheme which is analogous to that of Burke and Schumann [7] for laminar diffusion combustion.

Calculation of the diffusion jet can be reduced to the determination of the field of concentrations, temperatures, and velocities inside and outside the flame front and to the subsequent determination of the jet surface from the stoichiometric relation between the fuel and oxidant flows for which we have to solve the equations for the boundary layer (momentum, diffusion, and energy) under appropriate boundary conditions.

For an isobaric turbulent boundary layer the momentum, diffusion, and energy equation for ρu^2, $\rho u \Delta C$, and $\rho u \Delta T^*$ can be transformed by the introduction of new variables to equations of parabolic type [6, 8].

It is easy to show that in the case of a nonisobaric flow (when there is mixing or combustion in the chamber) we can similarly obtain equations of a parabolic type for $\rho + \rho u^2$, $\rho u \Delta C$, and $\rho u \Delta T^*$. The usual assumptions are made that the pressure depends only on the longitudinal coordinate and the wall friction can be neglected. Thus, for flow in a cylindrical chamber we have

$$\frac{\partial \Phi_k}{\partial \xi_k} = \frac{1}{R} \frac{\partial}{\partial R}\left(R \frac{\partial \Phi_k}{\partial R}\right), \quad k = \text{i, c, t.} \tag{1}$$

Here $\Phi_i = p + \rho u^2$, $\Phi_c = \rho u \Delta C$, $\Phi_t = \rho u \Delta T^*$, while ξ_k is a new variable whose relation to the coordinate x is determined experimentally. For isobaric flows $\sqrt{\xi_k} = C_k x$, where the coefficient of proportionality determines the intensity of mixing and depends on the parameters of the intermingling flows [8].

The solution of these equations in the coordinates ξ_k and R, for given boundary conditions, is easily obtained explicitly by known methods. Subsequent algebraic transformations (using the equation of state)

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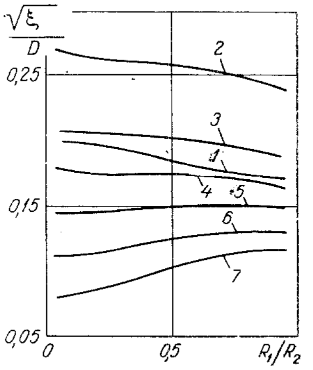

Fig. 1. The nondimensional flame length as a function of R_1/R_2 for various values of the coefficient of air excess: 1) $\alpha_\Sigma = 0.75$; 2) 1.1; 3) 1.5; 4) 2.0; 5) 3.0; 6) 5.0; 7) 10.0.

make it possible to pass from the Φ_k to the values of the velocity, concentration, and temperature and thus complete the calculation.

But if we are interested not in the complete pattern of the distribution of the parameters in the jet, but only in the shape of the jet surface and the coefficient of completeness of combustion, there is a simpler way of solving the problem. In fact combustion introduces no changes in the dynamical and diffusion equation of the boundary layer if the latter is written not in the true concentrations, but in the atomic [9] or reduced concentrations [1]. The atomic or reduced concentration is a conservative (conservable) quantity which is not affected by combustion and so the diffusion equation for flows of reduced concentrations (or linear combinations thereof) have the same form when there is combustion as when there is pure mixing and are valid for the whole volume of the chamber.

If we introduce a linear combination of flows of reduced concentration of fuel C_f and oxidant C_o, defined thus: $Z = \rho u(C_f - (C_o/L))$, then from (1) we obtain a similar equation for Z

$$\frac{\partial Z}{\partial \xi_c} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial Z}{\partial R}\right). \tag{2}$$

For the case of a single coaxial jet in a cylindrical chamber the boundary conditions have the form

$$Z = \begin{matrix} \rho_1 u_1 C_{f1} & \text{for} & 0 \leqslant R \leqslant R_1 \\ -\dfrac{\rho_2 u_2 C_{02}}{L} & & R_1 \leqslant R \leqslant R_2 \end{matrix}; \quad \frac{\partial Z}{\partial R} = 0 \quad \text{for} \quad \begin{matrix} R = 0 \\ R = R_2 \end{matrix}. \tag{3}$$

Noting (3), the solution of equation (2) has the form

$$Z = (1 - \alpha_\Sigma) f + \left(1 + \alpha_\Sigma \frac{f}{1-f}\right) F(\xi_c, R). \tag{4}$$

Here $\alpha_\Sigma = \rho_2 u_2 C_{02}(1-f)/\rho_1 u_1 C_{f1} f L$ is the coefficient of air excess and

$$F(\xi_c, R) = \sum_{i=1}^{\infty} \frac{2}{\mu_i}\frac{R_1}{R_2}\frac{J_1\left(\mu_i \frac{R_1}{R_2}\right)}{J_0^2(\mu_i)} J_0\left(\mu_i \frac{R}{R_2}\right)\exp\left(-\mu_i^2 \frac{\xi_c}{R_2^2}\right), \tag{5}$$

where the μ_i are the roots of the equation $J_1(\mu_i) = 0$.

Using (1) it is easy to define the shape of the surface and the flame length (Z = 0 on the flame front), and also the fuel burn-up curve in the coordinates ξ, R. Thus, the equation for the surface of the flame front is given by the implicit equation

$$F(\xi_c, R_f) = \frac{f(1-\alpha_\Sigma)}{1 + \alpha_\Sigma \dfrac{f}{1-f}}, \tag{6}$$

and the expression for the coefficient of completeness of combustion has the form

$$\eta_\Sigma = 1 - \frac{2}{R_1^2}\int_0^{R_f} \frac{\rho u C_f}{\rho_1 u_1 C_{f1}} R\,dR = 1 - \left(\frac{R_f}{R_2}\right)^2 (1 - \alpha_\Sigma)$$
$$+ \left(1 + \alpha_\Sigma \frac{f}{1-f}\right)\left(\frac{R_2}{R_1}\right)^2 \frac{R_f}{R_2} \sum_{i=1}^{\infty} \frac{4}{\mu_i^2} \frac{J_1\left(\mu_i \frac{R_1}{R_2}\right)}{J_0^2(\mu_i)} J_1\left(\mu_i \frac{R_f}{R_2}\right)\exp\left(-4\mu_i^2 \frac{\xi_c}{D^2}\right). \tag{7}$$

It follows from (6) that the shape and nondimensional length of the jet are defined primarily by the coefficient of air excess α_Σ. As α_Σ tends to unity the jet length tends to infinity.

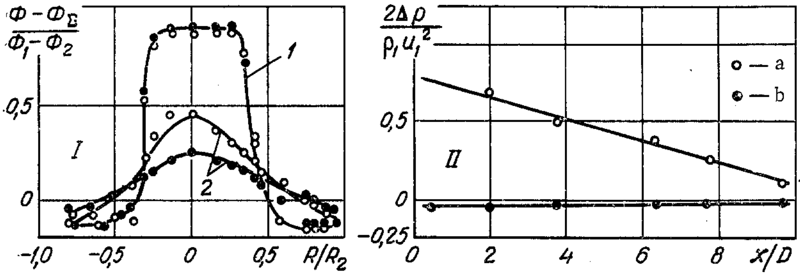

Fig. 2. Profiles of the relative excess impulse at various distances from the pipe [I, 1) x/d = 0.7; 2) 6.0] and the static pressure distribution along the chamber (II) for combustion (a) and for pure mixing (b):

u_1, m/sec	u_2, m/sec	T_1^*, °K	T_2^*, °K	α_Σ	m'
120	26,0	1073	425	1,15	0,34
120	25,5	1073	441	∞	0,345

The results of the calculations, shown on Fig. 1, show that for $\alpha_\Sigma \le 3$ the nondimensional jet length is practically independent of the ratio of the radius of the chamber to that of the pipe from which the gaseous fuel flows (in the range $0.05 \le R_1/R_2 \le 0.95$). In fact because of this, in what follows we can take not the diameter of the pipe, but the diameter of the combustion chamber as the defining dimension.†

Analysis of (7) leads to the important conclusion that the coefficient of completeness of combustion η_Σ is virtually a universal function of $(\sqrt{\alpha_\Sigma})(\sqrt{\xi_{cc}})/D$ for all values of α_Σ and R_1/R_2. Thus, for an air closed turbulent diffusion flame η_Σ depends on the length of the chamber, the coefficient of air excess α_Σ, and the conditions for turbulent mixing, which determine the form of the functional equation $\xi_{cc} = \xi_{cc}(x)$.

These theoretical conclusions naturally have to be verified experimentally. However, for the case of combustion in a cylindrical chamber we should first determine the form of the relation between the nondimensional coordinate ξ and the physical coordinate x. As our experiments showed, when we studied the mixing of turbulent nonisothermal coaxial flows in a cylindrical chamber for moderate longitudinal pressure drops, the function $\xi(x)$ has the same form‡ as for isobaric flows, i.e., $\sqrt{\xi_k} = C_k x$. As for isobaric flows,

$$\xi_c(x) = \xi_t(x) = \frac{\xi_i(x)}{\sigma}, \tag{8}$$

where σ is the analog of the Prandtl number (for the basic region, $\sigma \simeq 0.75$).

From an analysis of the results of these experiments in a chamber of diameter 150 mm, in which the following were varied: velocity and temperature of the central jet (u_1 = 100-530 m/sec, T_1^* = 873-1473°K), the velocity of the surrounding flow (u_2 = 10-140 m/sec, T_2^* = 400°K), the dimensions of the pipe (R_1 = 20-32 mm), and the initial intensity of the turbulence of the central jet (ε_1 = 3.5-7.2%), and also an analysis of the results of other investigations [11-14], the following equation for C_i can be proposed:

$$C_i = 0.035\left(\frac{\rho_2}{\rho_1}\right)^{0.15}(1-m'), \tag{9}$$

which holds for $0 \le m' \le 0.7$.

But (8) and (9) cannot automatically be extended to a flow with combustion. This is because when combustion takes place in the chamber a longitudinal pressure drop ensues of considerably greater magnitude than that in the case of pure mixing.

Under the action of the pressure drop and the combustion the fuel and air flows accelerate at different rates which can affect the intensity of their mixing and so change the form of Eqs. (8) and (9).

Hence, in the experimental investigation we have to determine the equation $\xi(x)$ for a flow with combustion and verify the theoretical relations obtained above.

† That the length of the jet is independent of the ratio R_1/R_2 has also been established for enclosed laminar diffusion flames [10].

‡ To determine the function we can, for example, compare the experimentally measured change in the impulse along the axis of the chamber $\Phi_{im} = f(x)$ with that computed theoretically $\Phi_{im} = f(\xi_i)$.

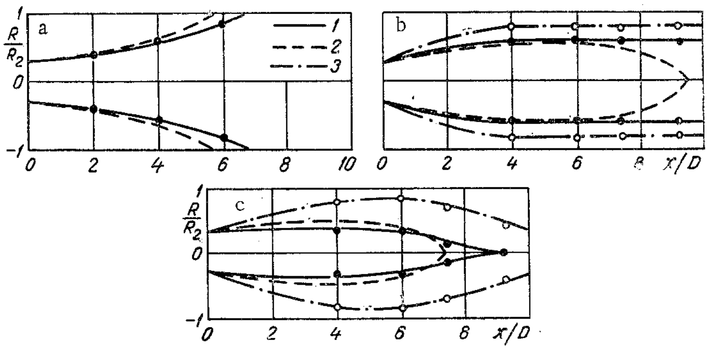

Fig. 3. The shape of the jet as a function of the coefficient of excess air [1, 2) experimental and theoretical surface of the flame front; 3) the surface where the local value of the completeness of combustion reaches 95%]:

c	α_Σ	u_1, m/sec	u_2, m/sec	T_1^*, K.	T_2^*, °K	m'	m'_c
a	0,71	230	11	1183	395	0,08	0,09
b	1,12	230	18,5	1183	403	0,14	0,25
c	2,62	235	40	1183	406	0,29	0,44

2. The experiments were made with two apparatuses which basically differed in the diameter of the combustion chamber. In the large apparatus a combustion chamber of diameter 150 mm was tested, while in the small apparatus interchangeable chambers of diameters 30, 40, and 60 mm were tested. In both cases the gaseous fuel was supplied through a central interchangeable pipe and air was supplied through an annular gap to the cylindrical chamber. In the fundamental series of experiments the pressure in the chamber was close to atmospheric. In the large apparatus the gaseous fuel was the products of the incomplete combustion of a benzine–air mixture (L = 1.8-2.4), while in the small apparatus it was a mixture of hydrogen and nitrogen (L = 4-34.5). A special pipe-burner with a pilot annular propane–oxygen flame (pipe diameters: 40, 56, and 68 mm) was used to stabilize the diffusion jet in the large apparatus; in the small apparatus, because the flow velocities were relatively small the jet was stabilized at the rims of the central pipe (pipe diameters 5, 10, and 15 mm).

In addition to the gaseous fuel and air consumptions and their parameters in the combustion chamber, we measured in the experiments not only the local but also the total coefficients of air excess and completeness of combustion of the fuel (by gas analysis), and also the field of total pressures and temperatures at various sections of the chamber and the distribution of static pressure along its length.

3. It is shown in Fig. 2 that the initial nonuniformity in the impulse field becomes smoothed out in the case of diffusion combustion and for pure mixing when the values of the parameters of the intermingling flows are the same at the entrance to the chamber. Comparison of these results shows that in the case of diffusion combustion the smoothing out of the impulse field is protracted, i.e., the intensity of turbulent mixing is reduced when there is combustion. But the form of the function $\xi_{ic}(x)$ when there is combustion is the same as for pure mixing, i.e.,

$$\sqrt{\xi_{ic}} = C_{ic}x, \quad (10)$$

and so the reduction in the intensity of turbulent mixing when there is combustion is sufficient to reduce the value of the coefficient C_{ic}.

A possible explanation of this effect is that the negative pressure gradient which occurs as a result of combustion accelerates the gaseous fuel and air flows to different degrees because these flows have different velocity heads (a low velocity head flow is accelerated to a greater degree than a high velocity head one). Also the relation between the velocity heads of the intermingling flows, which defines the intensity of turbulent mixing (cf. (9)), changes.

Analysis of the experimental results showed that the above divergence in the values of the coefficient C_i depends on the static pressure drop in the chamber. The coefficient C_{ic}, defining the intensity of mixing when there is combustion, in the first approximation can be determined from (9) by replacing m' with m'_c

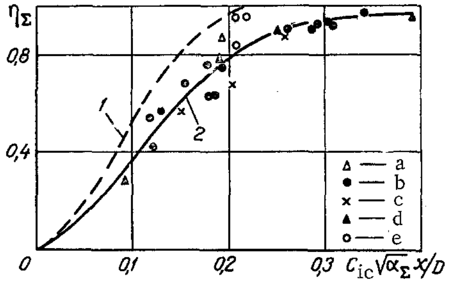

Fig. 4. The generalized universal relation between the completeness of combustion in the chamber and $C_{ic}\sqrt{\alpha_\Sigma x/D}$ [1) theoretical; 2) experimental]:

	a	b	c	d	e
α_Σ	0,71	1,12—4,46	1,0—4,6	1,1	1,0—4,5
L	2,37—2,46		4,0	7,4	35,0

$$m'_c = \frac{1}{2}\left\{m' + \sqrt{\frac{(m')^2 + 2\Delta p/\rho_1 u_1^2}{1 + 2\Delta p/\rho_1 u_1^2}}\right\}, \tag{11}$$

where $\Delta p/\rho_1 u_1^2$ is the nondimensional pressure drop in the chamber; for the one-dimensional flow of an incompressible gas

$$\frac{\Delta p}{\rho_1 u_1^2} = \frac{p_1 - p}{\rho_1 u_1^2}$$

$$= f\left\{\frac{(1+n)(1+n\theta+A)(1-f)}{\delta} - 1 - \frac{n^2\theta f}{\delta(1-f)}\right\}. \tag{12}$$

Thus, Eqs. (8)-(12) make possible the transition from the (ξ, R)-plane to the physical (x, R)-plane both in the case when there is combustion and when there is pure mixing.

In the experimental investigation the front of the diffusion flame was determined as the surface on which the local values of the coefficient of excess air were equal to unity (α = 1.0). The experiments showed (cf. Fig. 3) that, in accordance with theoretical considerations, for $\alpha_\Sigma < 1.0$ the flame front is fixed at the wall of the combustion chamber, while for $\alpha_\Sigma > 1.0$ it is fixed at its axis; for $\alpha_\Sigma \simeq 1.0$ the jet is sharply lengthened. But combustion is not complete at the flame front; it takes place in a quite extended zone the width of which increases with distance from the jet root (to illustrate this Fig. 3 shows the surface corresponding to η = 0.95). In the region external to the surface of the front we find fuel, while in the interval region we find oxygen.

Thus, the accepted quasilaminar model of a turbulent diffusion jet is an approximate one. The reasons for the divergence between the model and the actual pattern of the process of combustion can apparently be connected either with the effect of turbulent pulsations causing oscillations of the flame front about a mean position [1] or with a finite rate for the chemical reactions [9]. As a result of special experiments it was established that changes in the absolute velocities of both flows (from u_Σ = 140 to 280 m/sec for α_Σ = 1.7) or in the pressure in the chamber (from 1 to 2 bar) had no effect on the distribution of the local values of α and η at various cross sections of the chamber, i.e., had no effect on the burn-up of the fuel. From this result we can assume that the appearance of an extended combustion zone when the diffusion jet is safely stabilized is determined not by the finite rate of the chemical reactions, but by the characteristics of turbulent mixing, which, in the analogous regions of the flow, does not depend on the absolute values of the parameters of the flows.

In spite of the approximate nature of the scheme under discussion, it can explain, to an adequately good approximation with the results of the experiments, the relation between the coefficient of completeness of combustion of the fuel and the various parameters. For example, the reduction in η_Σ at constant α_Σ when the temperature of the air entering the chamber rises or when the diameter of the central pipe is increased is explained in the first case by an increase in the ratio u_2/u_1 and a reduction in the ratio ρ_2/ρ_1, and in the second case by an increase in u_2/u_1, i.e., is due to a deterioration in the mixing process.*

At the same time if the ratio of the velocities of the intermingling flows is kept constant, the completeness of combustion of the fuel in the chamber rises as α_Σ increases, since the length of the diffusion jet decreases (cf. Fig. 1). In these experiments with the small apparatus α_Σ was varied by changing the diameter of the combustion chamber.

In order to explain the effect of the stoichiometric coefficient on the diffusion combustion process, experiments were made with the small apparatus using a mixture of hydrogen and nitrogen. As was to be expected, at constant α_Σ and with identical mixing conditions, the completeness of combustion of the fuel was virtually independent of the stoichiometric coefficient.

A generalization of the experimental results of the experiments with both apparatuses is given in Fig. 4 in the form of a relation between the coefficient of the completeness of combustion of the fuel and

*An increase in the air temperature from 287 to 493°K leads to a reduction in η_Σ by 5% for x/D = 8 and α_Σ = 1.7; an increase in the diameter of the central pipe from 40 to 68 mm leads to a reduction in η_Σ from 0.97 to 0.77 for x/D = 11 and α_Σ = 1.7.

$C_{ic}\sqrt{\alpha_\Sigma x/D}$. We recall that it follows from theoretical considerations, taking note of (10), that this relation is universal. As we see, the experimental points are indeed grouped about a single curve which is similar to the theoretical curve, but lies somewhat below the latter. The reason for the difference between the curves is clear from the foregoing and is connected with the approximate nature of the quasilaminar model of the turbulent diffusion flame assumed in the theoretical analysis.

The generalized universal equation we have obtained $\eta_\Sigma = f(C_{ic}\sqrt{\alpha_\Sigma x/D})$ makes it possible to determine the completeness of combustion of the fuel in a chamber of given geometry if we know the coefficient of excess air and the parameters of the flows at the inlet to the chamber.

It should be emphasized that the above relation is valid only for conditions in which the diffusion flame is safely stabilized right at the end of the pipe through which the gaseous fuel enters the combustion chamber.

NOTATION

$A = Hu\eta_\Sigma / c_p T_1^*$	is the relative heat generation in the chamber;
C	is the concentration by weight;
C_i	is the coefficient determining the intensity of the turbulent exchange;
$D = 2R_1$	is the diameter of the chamber;
$f = R_1^2/R_2^2$	is the relative area of the central pipe;
J_0, J_1	are the Bessel functions of the first kind;
$m' = \sqrt{\rho_2 u_2^2/\rho_1 u_1^2}$	is the square root of the ratio of the velocity heads;
L	is the stoichiometric coefficient;
x	is the longitudinal coordinate;
R	is the radius;
α	is the coefficient of excess air;
$\delta = \mu_2/\mu_1$	is the ratio of the molecular weights;
η	is the coefficient of completeness of combustion;
$\theta = T_2^*/T_1^*$	is the stagnation temperature ratio;
$\eta = \rho_2 u_2 (1 - f)$ $/\rho_1 u_1 f$	is the flow-rate ratio;
ξ	is the nondimensional longitudinal coordinate;
ΔT^*, ΔC^*	are the stagnation temperature and concentration differences, respectively;
ρ, u, p, T	are the current density, velocity, pressure, and temperature.

Subscripts

1, 2	denote the parameters of the central and surrounding flows at the entrance to the chamber;
c	denotes the combustion case;
fr	denotes the flame front;
Σ	denotes all the parameters.

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